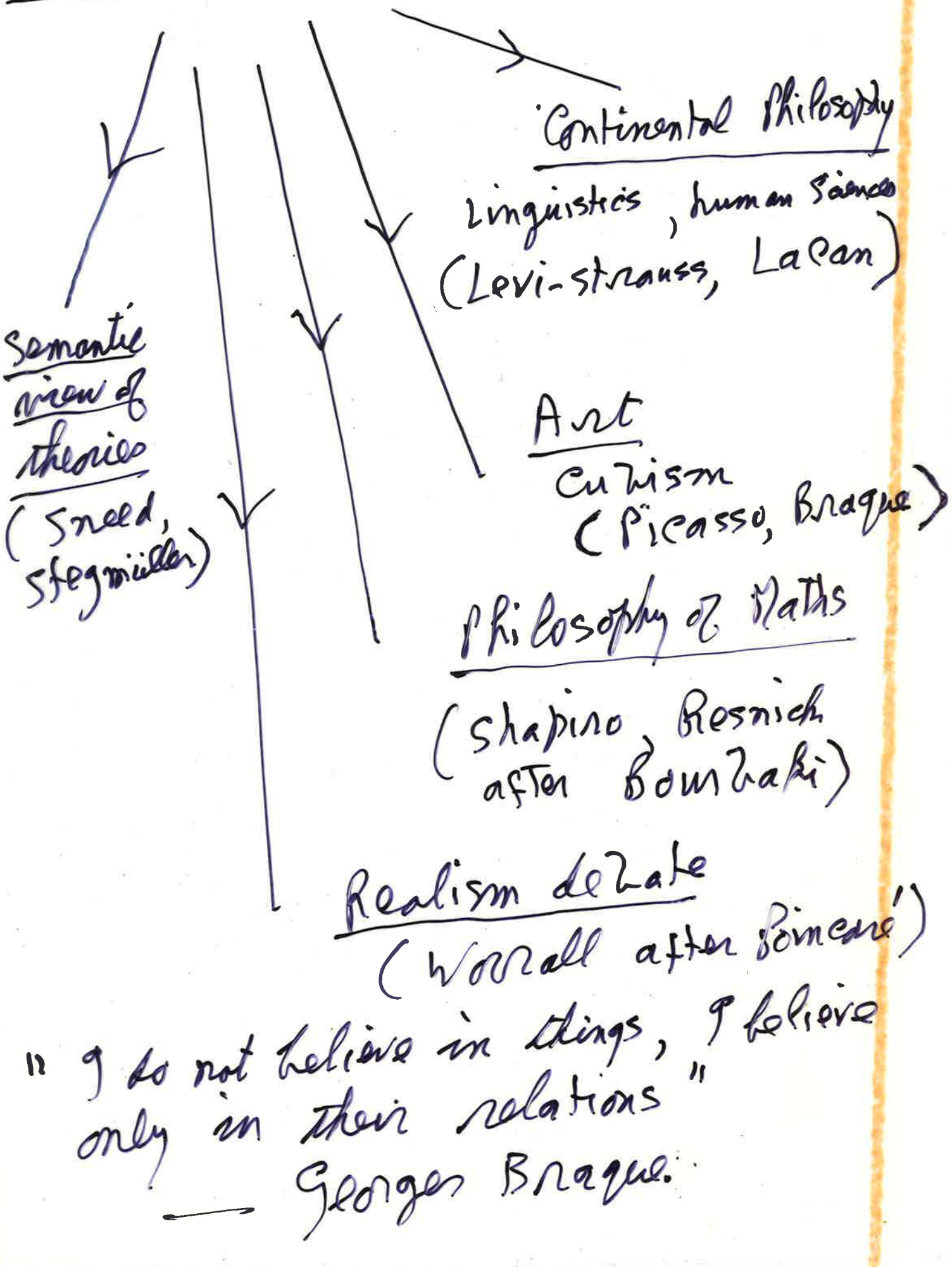
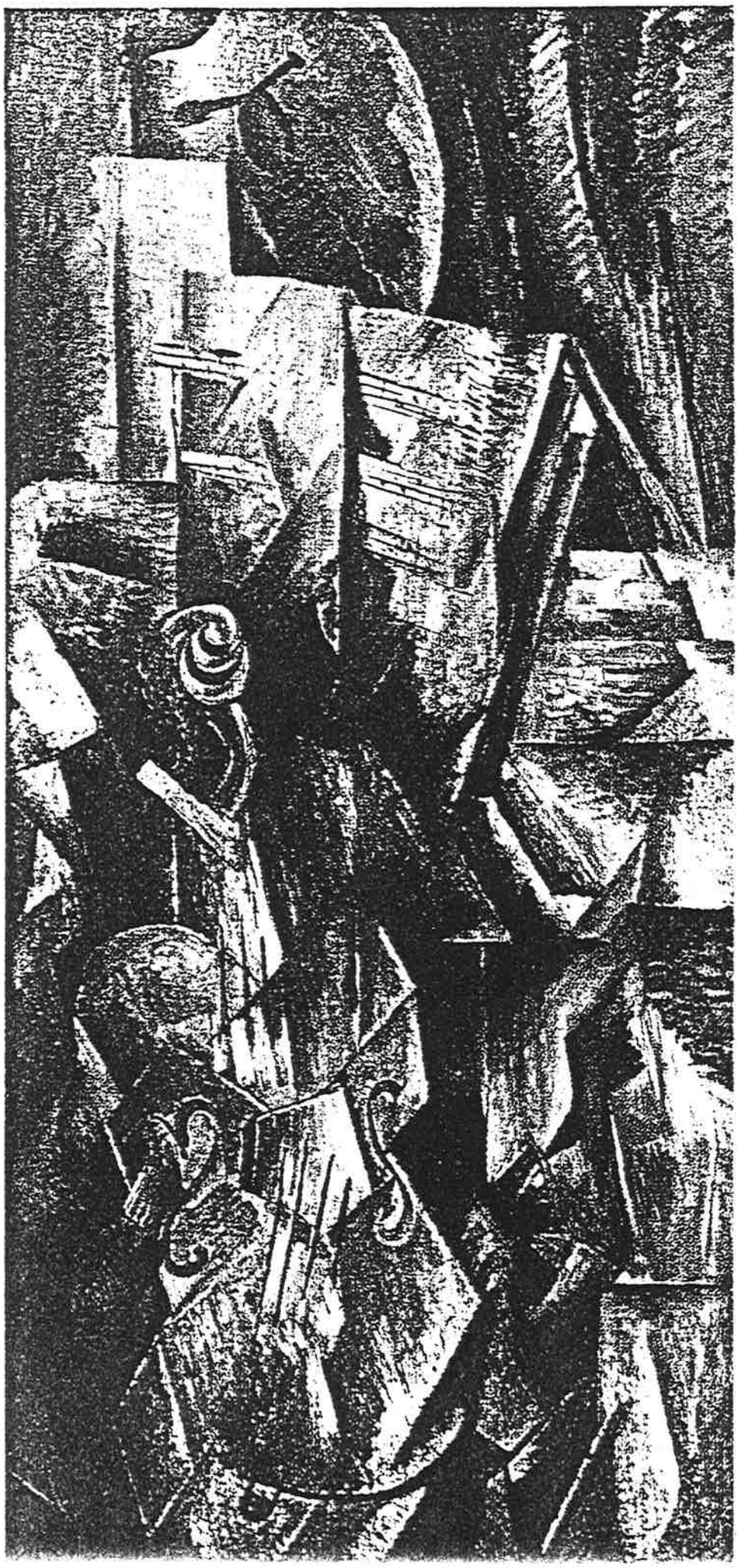
STRUCTURALISM



" I do not believe in things, I believe only in their relations "

— Georges Braque.

"Violin and Palette," oil painting by Georges Braque, 1910. In the Solomon R. Guggenheim Museum, New York City. 93 × 43 cm.



What is a structure?

The form of a possible system of related objects, ignoring the features of the objects that are not relevant to the interrelations

— Shapiro

LO. relations abstracted from the objects

cp. What *is* an electron? with the question, what are the equations governing its behaviour?

Queries. (1) Can the structure define the objects?

(2) Can the structure distinguish the objects?

(3) Can we be realists about the structures and anti realists about the objects?

structure of a Relation

R

Equivalence class of isomorphically related relations R and R' :

$R(x_1 \ldots x_n)$ iff $R'(y_1 \ldots y_n)$

where $y_1 \ldots y_n$ are images of $x_1 \ldots x_n$ under the isomorphism

N.B. This is a concept of abstract structure as contrasted with a concrete structure, a set of elements with relations defined on it. (what Shapiro calls a system)

Newman's Theorem

$\exists R\,(S(R))$ is a logical truth, modulo the cardinality of the domain over which the relations are defined.

Ex Consider Universe with 2 individuals $\underline{a}$ and $\underline{b}$

Write $1 = \langle a, b\rangle$, $2 = \langle b, a\rangle$, $3 = \langle a, a\rangle$ and $4 = \langle b, b\rangle$

and denote $\{1\}$ by 1, $\{1,2\}$ by 12, etc.

Then there are 16 binary relations defined on $\{a, b\}$ and 10 structures.

1234

123 134 234 124

12 13 14 23 24 34

1 2 3 4

∅

Relations	structures
1	1
4	2
6	4
4	2
1	1
16	10

Now choose, for example, the structure $S = \{1, 2\}$. Then $S(R)$ is satisfied by relations 1 or 2, and this is true whatever structure S we choose, since <u>all</u> relations are included in the diagram.

<u>But</u> if $\overline{R}$ is a <u>given</u> relation then $\overline{R}$ determines S, but S does not in general determine R.

<u>So</u> if we are only given S do we ever know what relation we are talking about?

Compare

Ontological structuralism

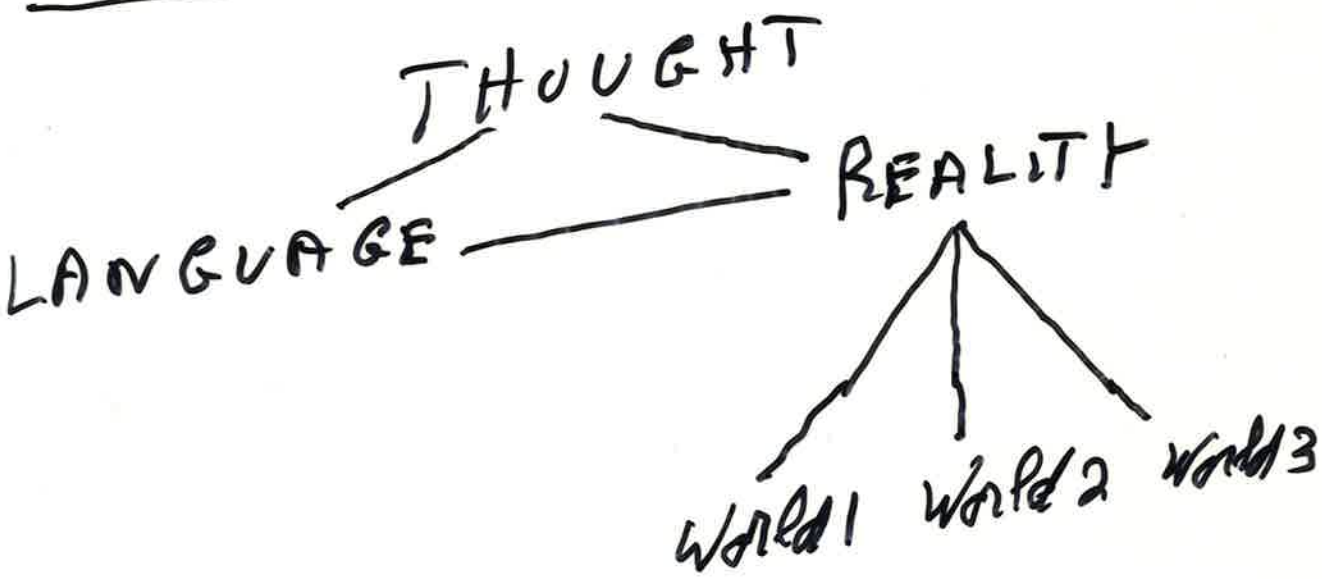
R does not exist, only S!

and

Epistemological structuralism

R exists but we only know S

Philosophy of Language

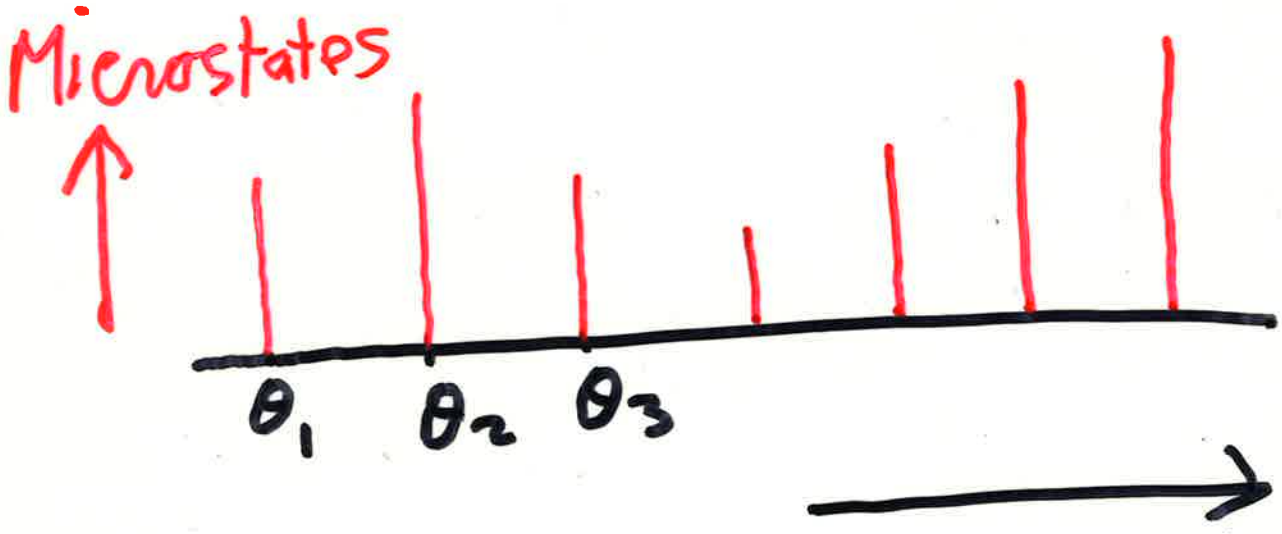


One Way to Go

$\bar{R}$ is an ontological posit,
a conjecture about world,
not necessarily a true description
of it, a denizen of world 3
if you will.

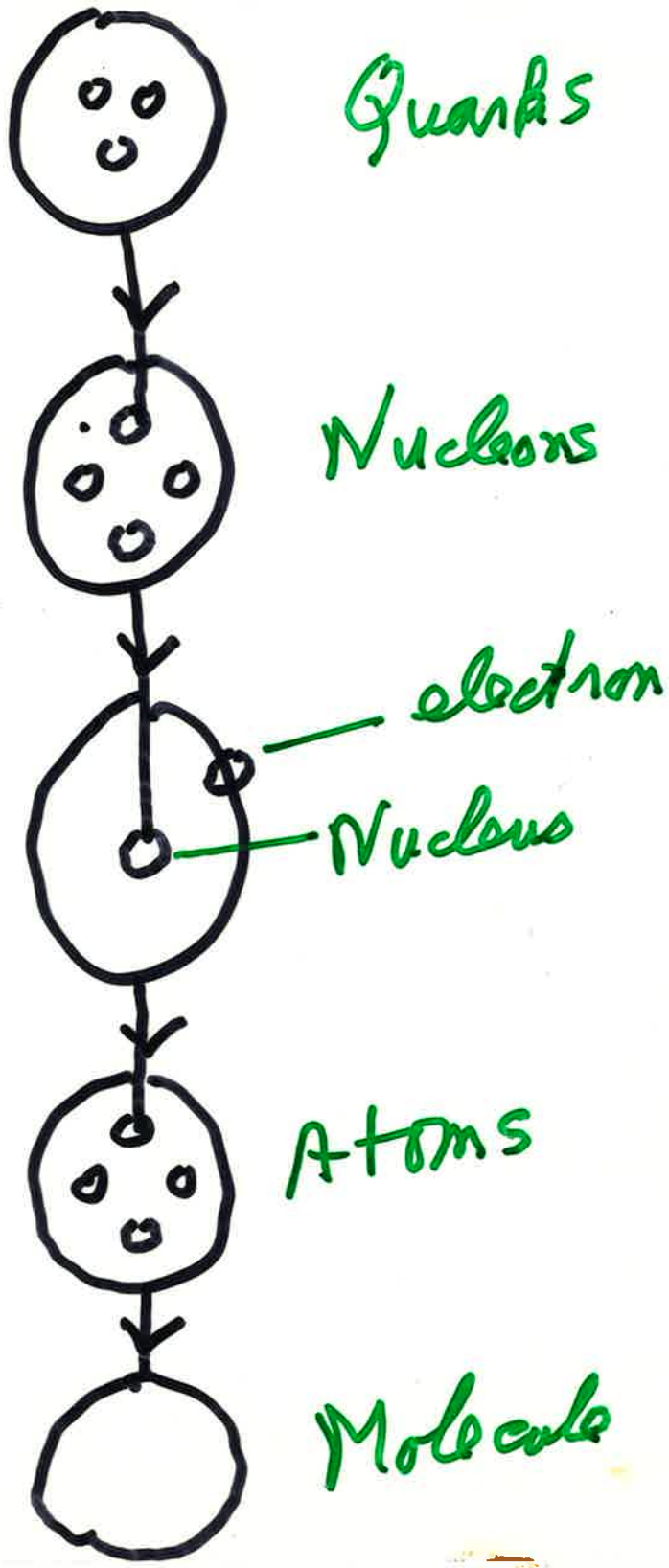
What aspect of $\bar{R}$ do we Realist
warrant to believe?
The claim is that it is the (abstract)
structure S that it satisfies.

Expanding and Contracting Structures



Each thermodynamic state can be 'expanded' into the collection of associated microstates.
Conversely, each vertical line can be 'contracted' (ie. projected) onto the corresponding thermodynamic state.

'Chinese Boxes' model of successive micro-reductions



Stability of structures

$\{S_p\}$ is family of structures

Mode of comparison of structures is provided by a transformation group T

If $\exists\, t_{p'p} \in T$ s.t. $S_{p'} = t_{p'p} S_p$

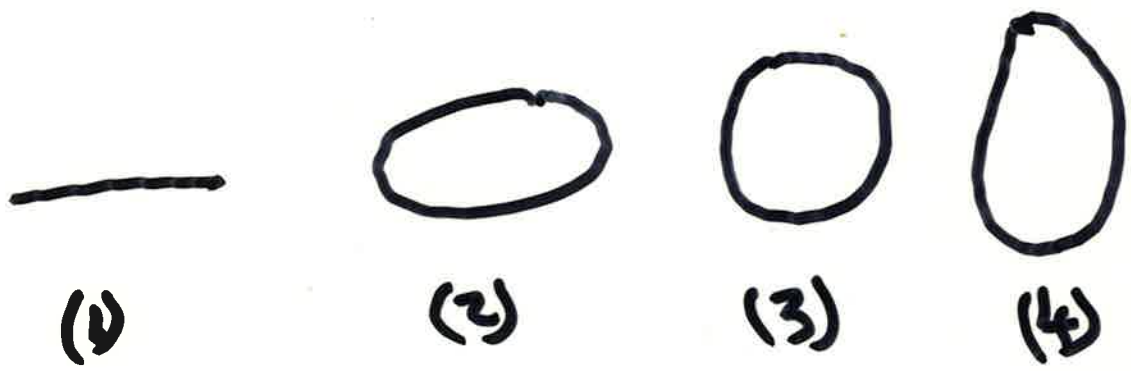
we say that $S_{p'}$ and S_p are structurally stable relative to T and write $S_{p'} \sim S_p$

If $S_p \sim S_{p'}$, $\forall p, p' \neq 0$

but $S_p \not\sim S_0$, $\forall p \neq 0$

we say that S_0 is singular

Ex Let T be the group of homeomorphisms of the Euclidean plane, then in diagram below, structures (2), (3) and (4) are structurally stable, but (1) is singular



Light-cone structure of special relativity

$1/c = 0$ ⟶ Increasing values of $1/c$

For $\frac{1}{c} = 0$, the metric of space-time $\begin{pmatrix} 1 & & & \\ & -1/c^2 & & \\ & & -1/c^2 & \\ & & & -1/c^2 \end{pmatrix}$ becomes singular and Minkowski space-time degenerates into Galilean space-time

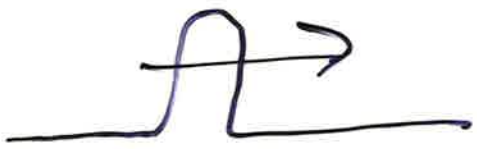
The limit of the wave equation as $1/c \to 0$

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \qquad (1)$$

becomes $$\frac{\partial^2\phi}{\partial x^2} = 0 \qquad (2)$$

Soln of (2) is $\phi = A(t)\,x + B(t)$

Unlike (1) solns of (2) cannot have compact support.

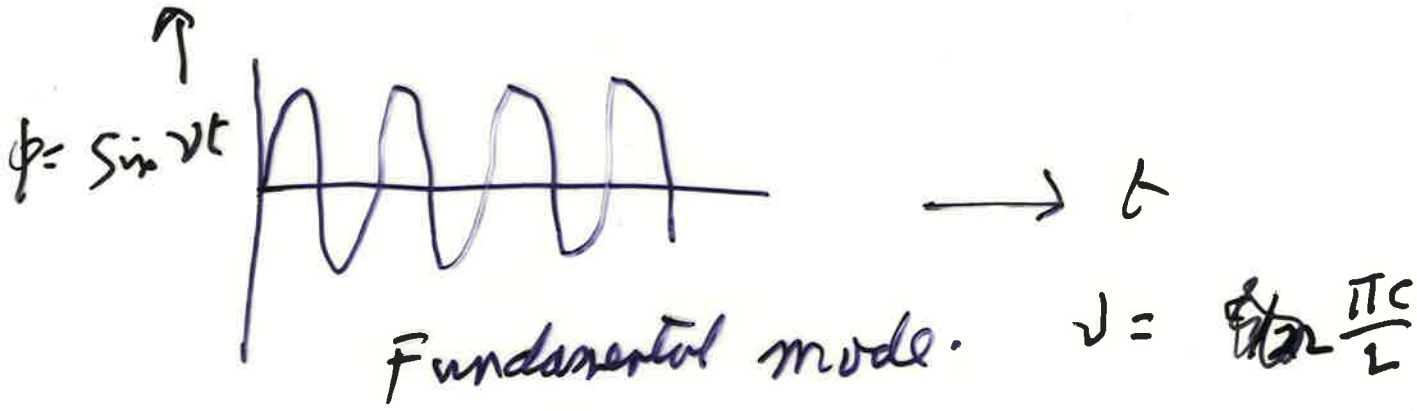
So wave pulse  cannot exist — zips instantly to ∞!

Consider motion of a string with boundary conditions $\phi = 0, \forall t$ at $x = 0, L$.

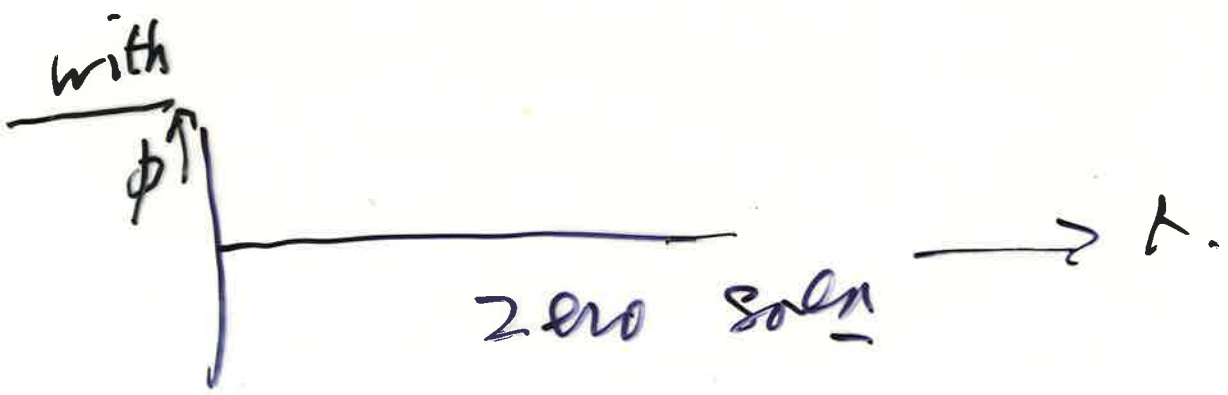
Soln of (2) is then just $\phi = 0$

But for any non-zero value of $1/c$ soln of (1) comprises pulses travelling to right and left and reflected at each end with 180° change in phase (D'Alembert's solution)

Compare



Fundamental mode. $\nu = \frac{\pi c}{L}$



Over time interval T, average of ϕ is

$$\bar{\phi} = \frac{1}{T}\int_0^T \sin \nu t \, dt = \frac{1}{\nu T}(1 - \cos \nu T)$$

$$= \frac{2L}{\pi c T}(1 - \cos \nu T)$$

For given T, $\bar{\phi} \to 0$ as $c \to \infty$

but for any c, we can always choose a T so that $\bar{\phi} \neq 0$.

So we have convergence of the mean rather than pointwise convergence.